



# Electrical Science

## DC MACHINES

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# D.C. Machines

## D.C. Generator

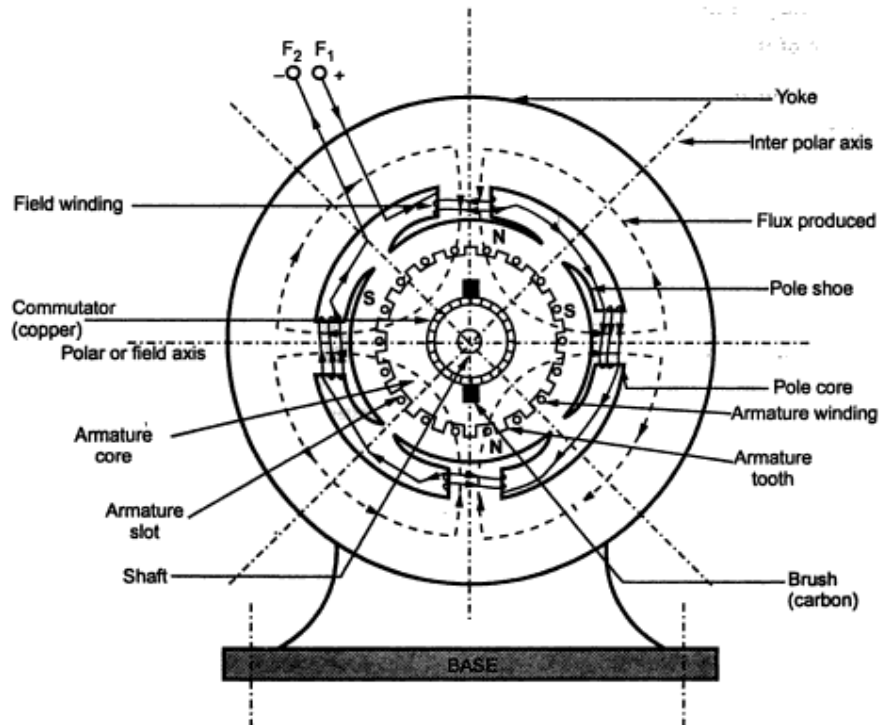
A D.C. machine which converts mechanical energy into electrical energy is known as **Generator**.

### Working Principle

The production of electromotive force is based on Faraday's laws of Electromagnetic induction. When a conductor or a coil is rotated in a magnetic field by the prime mover, it cuts magnetic lines of force and hence an e.m.f. is induced in the conductor or coil. Since there is motion of the conductor, it is known as **dynamically induced e.m.f.** The e.m.f. induced in the conductor or coil is ALTERNATING. By connecting the ends of the coil to a device known as COMMUTATOR, e.m.f. in the external is D.C.

### Construction Of D.C. Generator

**Construction of D.C. Machine:** The end view of a 4 pole D.C. machine is shown in following figure.

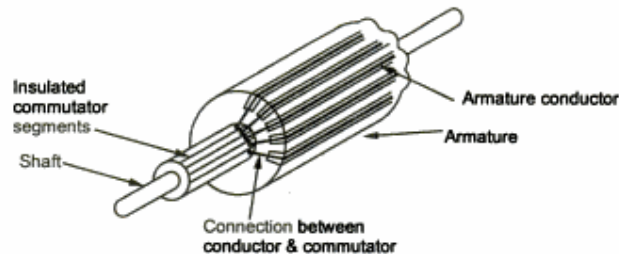


### Commutator

We have seen earlier that the basic nature of e.m.f. induced in the armature conductors is alternating. This needs rectification in case of d.c. generator, which is possible by a device called commutator.

- To facilitate the collection of current from the armature conductors.
- To convert internally developed alternating e.m.f. to unidirectional ( d.c.) e.m.f.
- To produce unidirectional torque in case of motors.

It is cylindrical in shape and is made up of wedge shaped segments of hard drawn, high conductivity copper. These segments are insulated from each other by thin layer of mica. Each commutator segment is connected to the armature conductor by means of copper lug or strip. This connection is shown in the following figure.



## **Brushes**

Brushes are stationary and resting on the surface of the commutator.

**Function.** To collect current from commutator and make it available to the stationary external circuit.

Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pig tail is used to connect the brush to the external circuit. To avoid wear and tear of commutator, the brushes are made up of soft material like carbon.

## **Armature**

It is further divided into two parts namely,

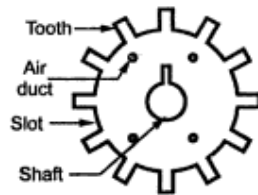
- Armature core and
- Armature winding

**Armature core:** Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

*Functions :*

- Armature core provides house for armature winding i.e. armature conductors.
- To provide a path of low reluctance to the magnetic flux produced by the field winding.

It is made up of laminated construction to keep eddy current loss as low as possible. A single circular lamination used for the construction of the armature core is shown in the figure.



**Armature winding:** Armature winding is nothing but the interconnection of the armature conductors, placed in the slots provided on the armature core periphery. When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f. gets induced in them.

*Functions*

- Generation of e.m.f. takes place in the armature winding in case of generators.
- To carry the current supplied in case of d.c. motors.
- To do the useful work in the external circuit.

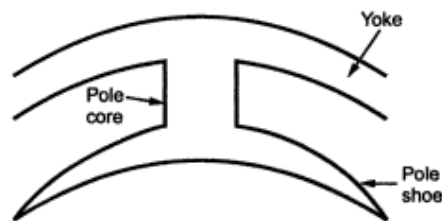
Armature winding is generally former wound. The conductors are placed in armature slots which are lined with though insulating material.

**Pole**

Each pole is divided into two parts namely,

- Pole core and
- Pole shoe

This is shown in the Fig.



*Functions of pole core and pole shoe :*

- Pole core basically carries a field winding which is necessary to produce the flux.
- It directs the flux produced through air gap to armature core, to the next pole.
- Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. To achieve this, pole shoe has been given a particular shape.

As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

The field winding is wound on the pole core with a definite direction.

**Functions:** To carry current due to which pole core, on which the field winding is placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as an electromagnet it is called Field winding or Exciting winding.

### Yoke

This is cast iron cast steel frame which provides mechanical protection and also acts as path for the magnetic flux shown by dotted lines.

### ARMATURE WINDINGS

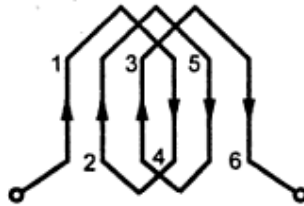
There are two types of windings

- Lap winding or Parallel winding
- Wave winding or two circuit winding or series wind

#### Lap winding

In this case, if connection is started from conductor in slot 1 then connections overlap each other as winding proceeds, till starting point is reached again.

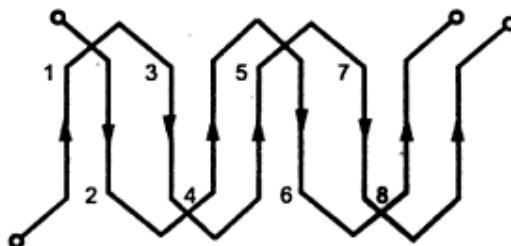
Developed view of part of the armature winding in lap fashion is shown in the Fig.



Due to such connection, the total number of conductors get divided into 'P' number of parallel paths, where  $P =$  number of poles in the machine.

#### Wave Winding

In this type of connection, winding always travels ahead avoiding overlapping. It travels like a progressive wave hence called wave winding. To get an idea of wave winding a part of armature winding in wave fashion is shown in the Fig.



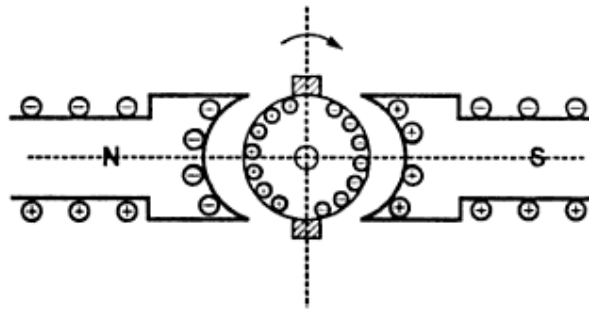
Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Due to this type of connection, the total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of the machine: As number of parallel paths are less, it is preferable for low current, high voltage capacity generators.

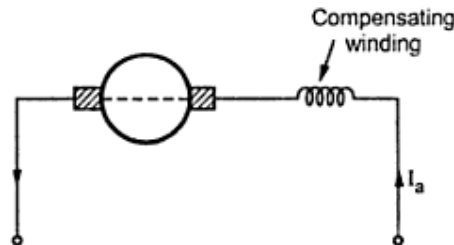
The number of parallel paths in which armature conductors are divided due to lap or wave fashion of connection is denoted as  $A$ . So  $A = P$  for lap connection and  $A = 2$  for wave connection.

### USE OF COMPENSATING WINDING

The compensating windings are basically used to neutralise the armature flux in the pole arc region which will otherwise cause severe distortion of main field flux. These windings are of concentric type and are placed in axial slots in the pole faces as shown in the figure.



The symbolic representation of compensating winding is shown in the following figure.



The armature reaction causes the displacement of main field flux. It affects the waveform of main field flux and makes it non-uniform. The effect of armature reaction depends upon armature current which in turn depends on the load on the machine.

In case of machines having large fluctuations in load such as rolling mill motors or turbo generators, the armature reaction will cause sudden shift of flux in backward and forward direction depending on change in the load. This will cause statically induced e.m.f. in the armature coils whose magnitude depends upon how fast the load is changing and by what amount it is changing. There is dynamically induced e.m.f. in the armature coil also. Under worst conditions these two e.m.f.s may become additive. This will occur when load is increased on motor and decreased from generator. If this e.m.f. is more than the breakdown voltage across adjacent commutator segments, a sparkover may occur which can easily

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spread over as conditions near commutator are favourable for flashover. The maximum allowable voltage between the segment is 30 to 40 V. Thus there is always danger of short circuiting the whole armature if armature flux is not compensated.

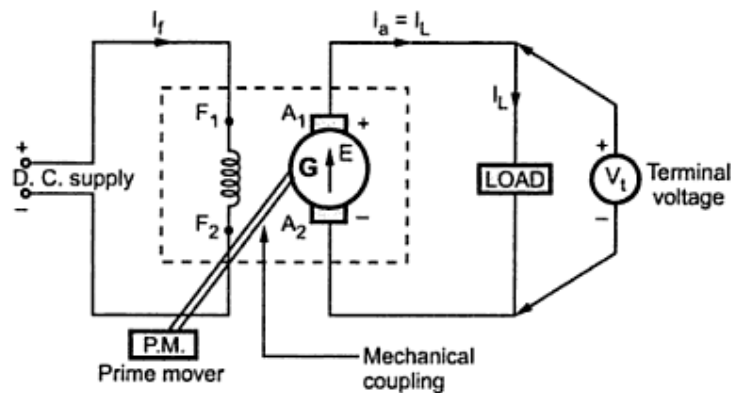
This can be achieved by the use of compensating winding which will neutralize the effect of armature reaction. These windings are connected in series with the armature. The current in these windings flows in opposite direction to that in armature conductors below the pole shoes. This will counterbalance the cross magnetising effect of armature reaction which may cause flashover between the segments.

**TYPES OF D.C. GENERATOR**

1. Separately excited D.C. Generator
2. Self excited
  - (a) D.C. Series Generator
  - (b) D.C. Shunt Generator
  - (c) D.C. Compound Generator
    - i. Long shunt
    - ii. Short shunt

**Separately Excited D.C. Generator**

In separately excited D.C. generator, there is no connection between the armature and field winding. To produce flux the field winding is connected to D.C. supply.

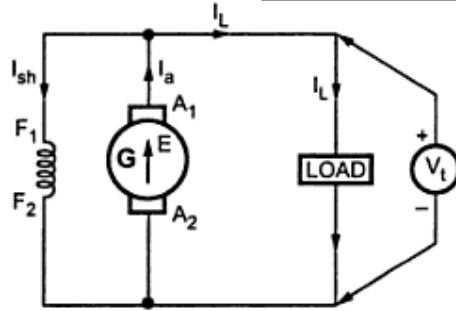
**Self excited D.C. Generator**

Classification of these generator is based upon the manner in which field winding is connected to the armature.

**D.C. Series Generator**

When the field winding is connected in parallel with the armature and the combination across the load then the generator is called shunt generator.

The field winding has large number of turns of thin wire so it has high resistance. Let  $R_{sh}$  be the resistance of the field winding.



Now  $I_a = I_L + I_{sh}$

Now voltage across load is  $V$ , which is same across field winding as both are in parallel with each other.

$$I_{sh} = \frac{V_t}{R_{sh}}$$

While induced e.m.f.  $E$ , still requires to supply voltage drop  $I_a R_a$  and brush contact drop.

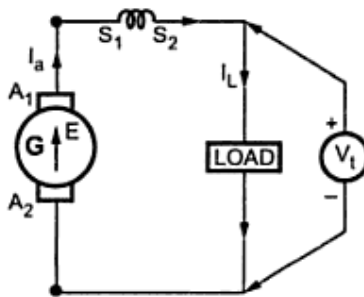
$$E = V_t + I_a R_a + V_{brush}$$

where  $E = \frac{\phi P N Z}{60 A}$

*In practice, brush contact drop can be neglected.*

### D.C. Shunt Generator

When the field winding is connected in series with the armature winding while supplying the load then the generator is called series generator. It is shown in the following Fig.



Field winding, in this case is denoted as  $S_1$  and  $S_2$ . The resistance of series field winding is very small and hence naturally it has less number of turns of thick cross-section wire as shown in the given figure.

Let  $R_{se}$  be the resistance of the series field winding.

As all armature, field and load are in series they carry the same current.

$$I_a = I_{se} + I_L$$

where  $I_{se}$  = Current through series field winding.



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Now in addition to drop  $I_a R_a$ , induced e.m.f. has to supply voltage drop across series field winding too. This is  $I_{se} R_{se}$  i.e.  $I_a R_{se}$  as  $I_a = I_{se}$ .

So voltage equation can be written as.

$$E = V_t + I_a R_a + I_a R_{se} + V_{brush}$$

$$\therefore E = V_t + I_a (R_a + R_{se}) + V_{brush}$$

where 
$$E = \frac{\phi P N Z}{60 A}$$

**E.M.F. EQUATION OF D.C. GENERATOR**

Let  $P$  = No. of poles

$\phi$  = Flux per pole in Weber

$Z$  = No. of armature conductors in series.

$N$  = No. of revolution per min.

$A$  = No. of revolution per min.

$t$  = No. of parallel path in the armature

When this conductor makes one revolution, total flux cut by it =  $P\phi$  Weber

If  $t$  is time of one revolution, e.m.f induced in one conductor =  $\frac{P\phi}{t}$

(Faraday's law of electro magnetic induction), e.m.f. induced in  $Z$  armature conductors.

$$\text{Connected in series} = \frac{P\phi Z}{60 / N} = \frac{P\phi N Z}{60} \text{ V}$$

This e.m.f. is induced in the closed circuit.

$$\text{e.m.f. available in external circuit} = \frac{P\phi N Z}{60 A} \text{ Volts}$$

$A$ , is the number of parallel path in the armature winding.

$A = P$  for lap connected armature

$A = Z$  for wave connected armature

**LOSSES IN A D.C. GENERATOR**

The various losses in a D.C. Generator are

**1. Armature copper losses =  $I_a^2 R_a$  watts (Variable)**

$I_a$  is the current in the armature and  $R_a$  is the resistance of armature winding. These losses are 30 to 40% of total losses.

**2. Copper losses in the field winding**

(i) **Shunt Generator** :  $I_{sh}^2 R_{th} = I_{sh}$  watts (constant)

These losses are constant.  $I_{sh}$  is shunt field current and  $R_{sh}$  is shunt field resistance.

(ii) **Series Generator**: These losses are variable.  $I_{se}^2 R_{se}$  watts.

$I_{sc}$  is the current in series field winding and  $R_{se}$  is the resistance of the series field winding.

Field copper losses are 20 to 30% of total losses. (Variable)

(iii) **Losses due to brush contact resistance** : These is some resistance between the brush and the commutator. The current passing from the commutator to brush is the armature current.

Therefore these losses are included in the armature copper losses.

(iv) **Magnetic losses or Iron losses**: These are divided into

(i) Hysteresis losses  $W_h \propto B_{max}^{1.6} f$

(ii) Eddy current losses  $W_e \propto B_{max}^2 f^2$

These losses are constant and are 20 to 30% of full load losses.

(v) **Mechanical losses**: These losses are divided into

(i) Friction losses at bearing and friction between commutator and brushes.

(ii) Air friction also known as winding losses of rotating armature.

These losses are 10 to 20 % of total losses.

3. **Stray losses**: Magnetic or Iron losses + Mechanical losses are known as stray losses.

4. **Constant losses**: Stray losses + Shunt Generator constant field losses are known as constant or Standing losses.

### **Total Losses in D.C. Generator**

= Armature copper losses + Field copper losses + Iron losses + Mechanical losses.

Efficiency of D.C. Generator

$$= \frac{\text{Output}}{\text{Input}} = \frac{VI_L}{VI_L + I_a^2 R_a + W_i} \quad (\text{Armature copper losses are } I_a^2 R_a)$$

$I_a = I_L + I_{sh}$ , Since  $I_{sh}$  is small and hence is neglected.

Therefore armature copper losses have been taken as  $I^2 R_a$ .

### **Example**

*A 4 pole, lap wound, d.c. generators has 42 coils with 8 turns per coils. It is driven at 1720 r.p.m. If useful flux per pole is 21 mWb, calculate the generated e.m.f. Find the speed at*

which it is to be driven to generate the same e.m.f as calculated above, with wave wound armature.

**Solution**

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$$P = 4, \phi = 21 \text{ mWb} = 21 \times 10^3 \text{ Wb}, N = 1120 \text{ r.p.m.}$$

$$\text{Coils} = 42 \text{ and turns/coil} = 8$$

$$\text{Total turns} = \text{coils} \times \text{turns / coil} = 42 \times 8 = 336$$

$$Z = 2 \times \text{total turns} = 2 \times 336 = 672$$

(i) For lap wound

$$A = P$$

$$E = \frac{\phi NZ}{60} = \frac{21 \times 10^{-3} \times 1120 \times 672}{60} = 263.424 \text{ V}$$

(ii) For wave wound

$$A = 2$$

$$E = 263.424$$

$$263.424 = \frac{21 \times 10^{-3} \times 4 \times N \times 672}{120}$$

$$\therefore N = 560 \text{ r.p.m.}$$

**Problem**

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A 4 pole d.c. generator has a wave wound armature with 792 conductors. If it is driven at a speed of 750 r.p.m., calculate the necessary flux per pole to generate 240 V on no load.

Answer: 12.12 mWb

**Problem**

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A 4 pole d.c. generator, having wave wound armature has 50 slots and 25 conductors per slot. Find the generated e.m.f. if it is driven at 25 r.p.s. and useful total flux in the machine is 0.03 Wb.

Answer: 468.75 V

Hint: There is no need to multiply given flux by number of poles as it is total flux and not the flux per pole.

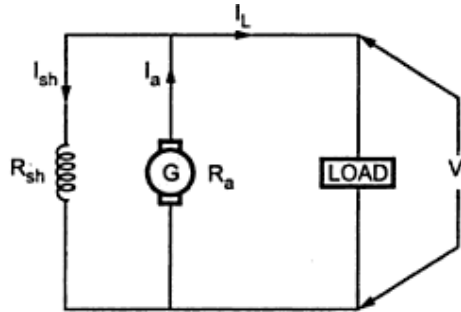
**Example**

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A d.c. shunt generator has shunt field winding resistance of 100  $\Omega$  It is supplying a load of 5 kW at a voltage of 250 V. If its armature resistance is 0.22  $\Omega$ , calculate the induced e.m.f. of generator.

Consider shunt generator as shown in the following figure.



We know  $I_a = I_L + I_{sh}$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{100} = 2.5A$$

Load power = 5 kW

$$P = V_t \times I_L$$

$$I_L = \frac{P}{V_t} = \frac{5 \times 10^3}{250} = 20A$$

$$I_a = I_L + I_{sh} = 20 + 2.5 = 22.5A$$

$$\therefore E = V_t + I_a R_a = 250 + 22.5 \times 0.22 = 254.95V \quad (V_{brush} \text{ is ignored})$$

This is the induced e.m.f. to supply the given load.

**Example**

The armature of a 4 pole, lap wound d.c. shunt generator has 40 coils with 8 turns per coil. Its shunt field resistance is  $70\Omega$  and armature resistance of  $0.03\Omega$ . If the flux per pole is 0.05 Wb. Find the speed of the machine when supplying 100 kW at a terminal voltage of 250V.

**Solution**

$$P = 4, \quad \phi = 0.05 \text{ Wb}$$

$$\text{Total turns} = \text{coils} \times \text{turns/coil} = 40 \times 8 = 320$$

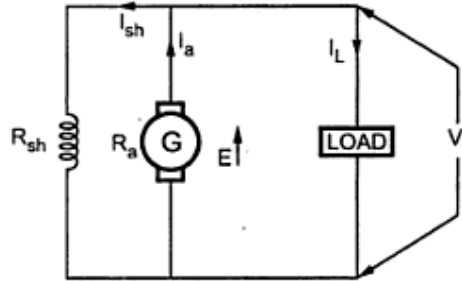
$$\text{Total conductors } Z = 2 \times \text{number of turns} = 2 \times 320 = 640$$

$$\text{Load power } P = 100 \text{ kW} = V_t I_L$$

$$100 \times 10^3 = 250 \times I_L$$

$$I_L = 400 \text{ A, as } V_t = 250 \text{ V}$$

Consider d.c shunt generator as shown in the following figure.



$$I_{sc} = \frac{V_t}{R_{sh}} = \frac{250}{70} = 3.333A$$

$$I_a = I_L + I_{sh} = 400 + 3.333 = 403.333A$$

$$E = V_t + I_a R_a = 250 + 403.333 \times 0.03 = 262.1V$$

Now 
$$E = \frac{\phi P N Z}{60 A}$$

Putting values  $E = 491.437 \text{ rpm}$

### Example

A 4 pole, lap wound shunt generator delivers 200 A at terminal voltage of 250 V. It has a field and armature resistance of 50  $\Omega$  and 0.05  $\Omega$  respectively. Determine: (i) Armature current, (ii) Current per armature parallel paths (iii) Generated e.m.f (iv) Power developed.

### Solution

$$P = 4, A = P = 4, I_L = 200 \text{ A}, V_t = 250 \text{ V}, R_a = 0.05 \Omega, R_{sh} = 50 \Omega$$

For shunt generator,

$$I_a = I_L + I_{sh}$$

and 
$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{50} = 5A$$

$$\therefore I_a = 200 + 5 = 205A$$

- (i) Armature current is 205 A.  
(ii) There are 4 parallel paths as lap connected armature winding.

$$\therefore \text{Current per parallel path} = 205/4 = 51.25 \text{ A}$$

- (iii) Generated e.m.f.  $E = V_t + I_a R_a$

$$E = 250 + 205 \times 0.05 = 260.25 \text{ V}$$

- (iv) Power developed =  $E \times I_a = 260.25 \times 205 = 53351 \text{ W}$

A 4 pole, lap wound 750 r.p.m. d.c. shunt generator has an armature resistance of  $0.4 \Omega$  and field resistance of  $200 \Omega$ . The armature has 720 conductors and the flux per pole is  $30 \text{ mWb}$ . If the load resistance is  $15 \Omega$ , determine the terminal voltage.

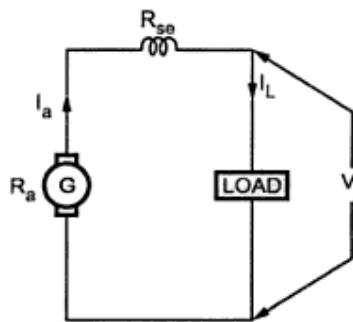
Answer:  $262.47 \text{ V}$

### Example

A dc series generator has armature resistance of  $0.5 \Omega$  and series field resistance of  $0.03 \Omega$ . It drives a load of  $50 \text{ A}$ . If it has 6 turns/coil and total 540 coils on the armature and is driven at  $1500 \text{ r.p.m.}$ , calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as  $2 \text{ mWb}$  and total brush drop as  $2 \text{ V}$ .

### Solution

Consider the series generator as shown in following figure.



$$R_a = 0.5 \Omega, R_{se} = 0.03 \Omega, V_{brush} = 2 \text{ V}$$

$$N = 1500 \text{ r.p.m.}$$

Total coils are 540 with 6 turns/coil.

$$\therefore \text{Total turns} = 540 \times 6 = 3240$$

$$\therefore \text{Total conductors } Z = 2 \times \text{turns} = 2 \times 3240 = 6480$$

$$\therefore E = \frac{\phi P N Z}{60 A}$$

For lap type,  $A = P$

$$\text{and } \phi = 2 \text{ mWb} = 2 \times 10^{-3} \text{ Wb}$$

$$E = \frac{2 \times 10^{-3} \times 1500 \times 6480}{60} = 324 \text{ V}$$

$$E = V_t + I_a (R_a + R_{se}) + V_{brush}$$

$$\text{where } I_a = I_L = 50 \text{ A}$$

$$\therefore 324 = V_t + 50(0.5 + 0.03) + 2$$

$$\therefore V_t = 295.5 \text{ V}$$

### EXAMPLE (AMIE, Summer 96)

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A shunt generator has an induced e.m.f. of 254 V. When the generator is loaded, the terminal voltage is 240 V. Neglecting armature reaction find the load current if the armature resistance is 0.04 ohm and the field circuit resistance is 24 ohms.

### SOLUTION

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Internal drop of shunt generator under load = 254 - 240 = 14 V

$$I_a R_a = 14 \quad \text{or} \quad I_a = \frac{14}{R_a} = \frac{14}{0.04} = 350 \text{ A}$$

$$\text{Field current under loaded condition} = \frac{240}{24} = 10 \text{ A}$$

$$\text{Load Current} = I_a - I_f = 350 - 10 = \mathbf{340 \text{ Amp.}}$$

### EXAMPLE (AMIE, Winter 96)

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A 100 kW, 460 V shunt generator was run as a motor on no load at its rated voltage and speed. The total current taken was 9.8 A, including a shunt current of 2.7 A. The resistance of armature circuit at normal working temperature was 0.11 ohms. Calculate the efficiency at full load.

### SOLUTION

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When machine runs as generator

$$I_L + \frac{(100 \times 10^3)}{460} = 2174 \text{ amp.}$$

$$I_f = 2.7 \text{ A}$$

$$\text{Hence, } I_a = 2174 - 2.7 = 214.7 \text{ amp.}$$

$$\text{Full load armature copper loss} = I_a^2 R_a = (214.7)^2 \times 0.11 = 5.07 \text{ kW}$$

When machine runs as motor at no load at rated voltage and speed, the no load power input = 460 x 9.8 = 4508 watts

$$\text{Armature current at no load} = 9.8 - 2.7 = 7.1 \text{ amp.}$$

$$\text{Armature copper loss} = (7.1)^2 \times 0.11 = 5.5 \text{ watts}$$

$$\text{Constant losses of machine} = 4508 - 5.5 = 4502.5 \text{ watts}$$

Total losses at full load of machine

$$= \text{Constant losses} + \text{Armature copper loss}$$

$$= 4502.5 + 5070 = 9572.5 \text{ kW}$$

%  $\eta$  of machine

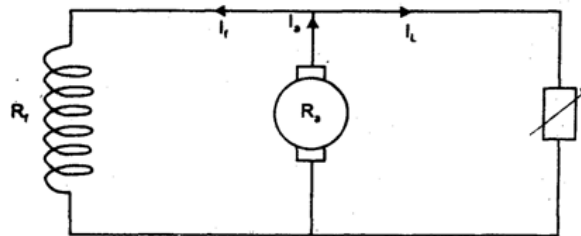
$$= \frac{\text{Output}}{(\text{Output} + \text{Losses})} \times 100 = \frac{100}{(100 + 9.5725)} \times 100 = \mathbf{91.25 \%}$$

**Example(AMIE Winter99, 8 marks)**

*A 4-pole shunt generator with lap connected armature has field and armature resistances of  $50\Omega$  and  $0.1\Omega$  respectively. It supplies power to sixty 100V, 40 W lamps. Calculate the total armature current, the current per armature path, and the generated electromotive force. Allow a contact drop of 1V per brush.*

**Solution**

See figure.



$$I_L = \text{Total load(Watts)/Terminal voltage(Volts)}$$

$$= (40 \times 60)/100 = 24 \text{ amp.}$$

Field current  $I_f = V/R_f = 100/50 = 2 \text{ amp.}$

Armature current

$$I_a = I_L + I_f = 24 + 2 = 26 \text{ amp.}$$

Number of parallel paths = number of poles = 4

$\therefore$  Current per path =  $26/4 = 6.5 \text{ amp.}$

Generated emf

$$= V + I_a R_a + \text{contact drop for two brushes}$$

$$= 100 + 26 \times 0.1 + 2$$

$$= 100 + 2.6 + 2 = 104.6 \text{ volts}$$



## D.C. Motors

A motor converts an electrical energy into mechanical energy, i.e. the motor is coupled to some mechanical load say a lathe.

### WORKING PRINCIPLE

When a current carrying conductor is situated in a magnetic field, it experience a mechanical force  $F = Bil$  Newton (B is flux density in  $wb/m^2$ , I is the current in amperes flowing through the conductor and l is the length of the conductor in meter) under the influence of this mechanical force, the conductor starts revolving. The direction of rotation is given by Fleming's left hand rule.

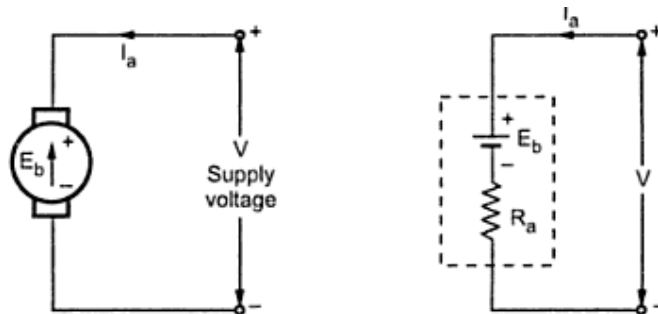
### SIGNIFICANCE OF BACK E.M.F.

It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f. gets induced in the conductor. The question is obvious that in a d.c. motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. So is there a generating action existing in a motor? The answer to this question is 'Yes'.

After a motoring action, there exists a generating action. There is an induced e.m.f. in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced e.m.f. in the armature always acts in the opposite direction of the supply voltage. This is according to the Lenz's law which states that the direction of the induced e.m.f. is always so as to oppose the cause producing it. In a d.c. motor, electrical input i.e. the supply voltage is the cause and hence this induced e.m.f. opposes the supply voltage. This e.m.f. tries to set up a current through the armature which is in the opposite direction to that, which supply voltage is forcing through the conductor.

So as this e.m.f. always opposes the supply voltage, it is called *back e.m.f.* and denoted as  $E_b$ . Though it is denoted as  $E_b$  basically it gets generated by the generating action which we have seen earlier in case of generators. So its magnitude can be determined by the e.m.f. equation which is derived earlier. So,

$$E = \frac{\phi PNZ}{60A} \text{ volts}$$



This e.m.f. is shown schematically in the Fig. (a). So if  $V$  is supply voltage in volts and  $R_a$  is the value of the armature resistance, the equivalent electric circuit can be shown as in the Fig. (b).

### **VOLTAGE EQUATION OF A D.C. MOTOR**

In case of a generator, generated e.m.f. has to supply armature resistance drop and remaining part is available across the load as a terminal voltage. But in case of d.c. motor, supply voltage  $V$  has to overcome back e.m.f.  $E_b$  which is opposing  $V$  and also various drops as armature resistance drop  $I_a R_a$ , brush drop etc. Infact the electrical work done in overcoming the back e.m.f. gets converted into the mechanical energy developed in the armature. Hence the voltage equation of a d.c. motor can be written as,

$$V = E_b + I_a R_a + \text{brush drop}$$

Neglecting the brush drop, the generalised voltage equation is,

$$V = E_b + I_a R_a$$

The back e.m.f. is always less than supply voltage ( $E_b < V$ ). But  $R_a$  is very small hence under normal running conditions, the difference between back e.m.f. and supply voltage is very small. The net voltage across the armature is the difference between the supply voltage and back e.m.f. which decides the armature current. Hence from the voltage equation we can write,

$$I_a = \frac{V - E_b}{R_a}$$

#### **Example**

---

*A 220 V, d.c. motor has an armature resistance of 0.75  $\Omega$  it is drawing an armature current of 30 A, driving a certain load. Calculate the induced e.m.f in the motor under this condition.*

#### **Solution**

---

$V = 220$  V,  $I_a = 30$  A,  $R_a = 0.75$   $\Omega$  are the given values.

For a motor,

$$V = E_b + I_a R_a$$

$$220 = E_b + 30 \times 0.75$$

$$E_b = 197.5 \text{ volts}$$

This is the induced e.m.f. called back e.m.f. in a motor.

#### **Example**

---

*A 4 pole, d.c. motor has top connected armature winding. The flux per pole is 30 mWb. The number of armature conductors is 250. When connected to 230 V d.c. supply it draws an*

armature current of 40 A. Calculate the back e.m.f. and the speed with which motor is running. Assume armature resistance is 0.6  $\Omega$ .

**Solution**

---

$$P = 4, A = P = 4 \text{ as lap, } V = 230 \text{ V, } Z = 250$$

$$4 = 30\text{mWb} = 30 \times 10^{-3} \text{ Wb}$$

$$L = 40 \text{ A}$$

From voltage equation,

$$V = E_b + I_a R_a$$

$$230 = E_b + 40 \times 0.6$$

$$E_b = \frac{\phi PNZ}{60A}$$

Putting various values

$$N = 1648 \text{ r.p.m.}$$

**Problem**

---

A 250V, d.c. shunt motor takes a line current of 20 A. Resistance of shunt field winding is 200  $\Omega$  and resistance of the armature is 0.3  $\Omega$  Find the armature current and the back e.m.f.

Answer: 18.75 A, 244.375 V

**BACK E.M.F. AS A REGULATING MECHANISM**

Due to the presence of back e.m.f. the d.c. motor becomes a regulating machine i.e. motor adjusts itself to draw the armature current just enough to satisfy the load demand. The basic principle of this fact is that the back e.m.f. is proportional to speed,  $E_b \propto N$ .

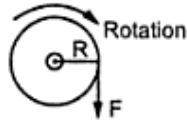
When load is suddenly put on to the motor, motor tries to slow down. So speed of the motor reduces due to which back e.m.f. also decreases. So the net voltage across the armature ( $V - E_b$ ) increases and motor draws more armature current. As  $F = B / I$ , due to increased current, force experienced by the conductors and hence the torque on the armature increases. The increase in the torque is just sufficient to satisfy increased load demand. The motor speed stops decreasing when the armature current is just enough to produce torque demanded by the new load.

When load on the motor is decreased, the speed of the motor tries to increase. Hence back e.m.f. increases. This causes ( $V - E_b$ ) to reduce which eventually reduces the current drawn by the armature. The motor speed stops increasing when the armature current is just enough to produce the less torque required by the new load.

So back e.m.f regulates the flow of armature current and it automatically alters the armature current to meet the load requirement. This is the practical significance of the back e.m.f.

## TORQUE EQUATION OF D.C. MOTOR

It is seen that the turning or twisting force about an axis is called torque. Consider a wheel of radius R meters acted upon by a circumferential force F Newton as shown in the Fig.



The wheel is rotating at a speed of N r.p.m.

Then angular speed of the wheel is,

$$\omega = \frac{2\pi N}{60}$$

So work done in one revolution is,

$$W = F \times \text{distance travelled in one revolution}$$

$$= F \times 2\pi R \quad \text{Joules}$$

$$P = \text{Power developed} = \text{work done/time}$$

$$= \frac{F \times 2\pi R}{\text{time for 1 rev}} = \frac{F \times 2\pi R}{60/N} = F \times R \left( \frac{2\pi N}{60} \right)$$

$$P = T \times \omega \quad \text{watts}$$

Where  $T$  = Torque in N – m,  $\omega$  = Angular speed in rad/sec.

Let  $T_a$  be the gross torque developed by the armature of the motor. It is also called armature torque. The gross mechanical power developed in the armature is  $E_b I_a$  as seen from the power equation. So if speed of the motor is N r.p.m. then.

$$\text{Power in armature} = \text{Armature torque} \times \omega$$

But 
$$E_b = \frac{\phi P N Z}{60 A}$$

$$\therefore \frac{\phi P N Z}{60 A} I_a = T_a \times \frac{2\pi N}{60}$$

$$\therefore T_a = \frac{1}{2\pi} \phi I_a \times \frac{P Z}{A} = 0.159 \phi I_a \cdot \frac{P Z}{A} \text{ N-m}$$

This is the torque equation of a d.c. motor.

### Problem

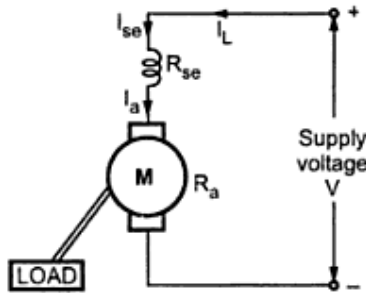
*A 4 pole d.c. motor takes a 50A armature current. The armature has lap connected 480 conductors. The flux per pole is 20m Wb. Calculate the gross torque developed by the armature of the motor.*

*Answer: 76.394 N-m*

1. D.C. series motor
2. D.C. Shunt motor
3. D.C. compound motor
  - (i) Short Shunt
  - (ii) Long Shunt

**SERIES MOTOR**

The connection diagram of D.C. series motor is shown in Figure.



Let  $I_L$  be the total current drawn from the supply

$$I_L = I_{sc} = I_a$$

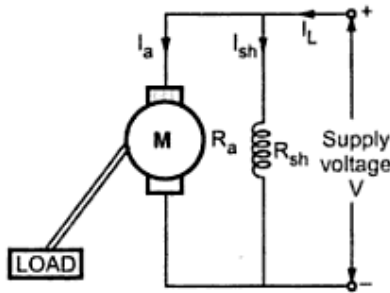
$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$= E_b (R_a + R_{se}) + V_{brush}$$

Supply voltage has to overcome the drop across series field winding in addition to  $E_b$  and drop across armature winding.

**D.C. SHUNT MOTOR**

The connection diagram of D.C. shunt motor is shown in figure .



From the figure

$$I_L = I_a + I_{sh} \tag{i}$$

$$I_{sh} = \frac{V}{R_{sh}} \quad (ii)$$

$$V = E_b + I_a R_a + \text{voltage drop at brushes}$$

$$E_b = V - I_a R_a \quad (\text{neglecting voltage drop at brushes}) \quad (iii)$$

Calculating the value of  $I_{sh}$  from equation (ii) and substituting its value in equation (i) we find the value of  $I_a$ .

By substituting the value of  $I_a$  in equation (iii) the value of  $E_b$  can be found.

## MOTOR CHARACTERISTICS

It is necessary to know the characteristics of different type of motors as the knowledge of the characteristics will help to choose a particular type of motor for driving the mechanical load. Load may be of variable speed type, constant speed type, low starting torque or of high starting torque. Knowledge of characteristics will decide the application of D.C. motors.

### Type of Characteristics

1. Torque versus armature current, i.e.  $T_a/I_a$ . These characteristics are also known as electrical characteristics.
2. Speed versus armature current i.e.  $N/I_a$ .
3. Speed versus torque, i.e.  $N/T_a$ . It is also known as mechanical characteristic. This  $N/T_a$  can be derived from the above two characteristics.

To explain the shape of the curves, the following equations are used.

1.  $T_a \propto I_a$
2.  $I_a = (V - E_b)/R_a$
3.  $N \propto E_b$

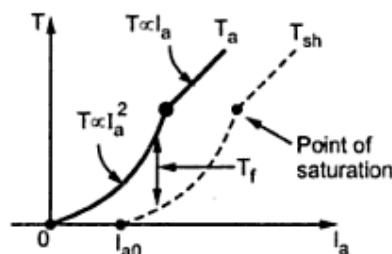
## D.C. Series Motor Characteristics

### $T_a/I_a$ Characteristics

From the torque equation of the motor  $T \propto I_a$ . Since the field is in series with the armature, therefore  $\phi \propto I_a$ .

$$T \propto I_a^2$$

Therefore, the shape is parabola.



**ELECTRICAL SCIENCE****D.C. MACHINES**

When the load current further increase so that the field becomes saturated. After saturation, flux  $\phi$  becomes constant therefore,  $T_a \propto I_a$ , therefore the curve is a straight line.

**Conclusion :** At the time of starting the D.C. Series motor  $T \propto I_a^2$ , therefore it is used where high starting torque is required i.e., hoists, electric traction, cranes etc.

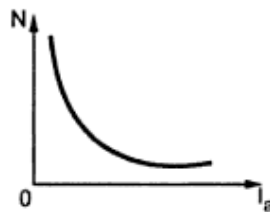
 **$N/I_a$  Characteristics**

From the equation  $N \propto (E_b/\phi)$

Change in  $E_b$  with change in load current is small and hence may be regarded constant.

$$N \propto 1/\phi.$$

Also  $\phi \propto I_a.$



Therefore as load current or armature current changes,  $\phi$  changes, the speed changes. At light loads or no load, the armature current is very small and hence the flux is very small. There the motor will attain a dangerously high speed. As the load current increases,  $\phi$  increases and hence speed decreases. The D.C. series motor is therefore variable speed motor; the speed decreases with increase in load current.

**Conclusion:**

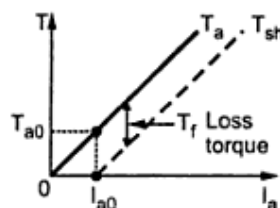
1. Since at no load, the speed is dangerously high D.C. series motor should never be run without load. It should not be belt driven.
2. It is a variable speed motor.
3. It has high starting torque.

 **$N/T_a$  Characteristics**

The shape of the curve is similar to  $N/I_a$  characteristics.

**D.C. Shunt Motor Characteristics** **$T_a/I_a$  Shunt Motor Characteristics**

$$T \propto \phi I_a.$$



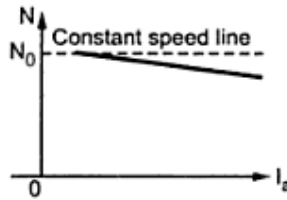
**ELECTRICAL SCIENCE****D.C. MACHINES**

Since in a shunt motor, field current  $I_{sh}$  is constant and flux is proportional to  $I_{sh}$  therefore flux  $\phi$  is constant. Therefore  $T \propto I_a$ , hence  $T/I_a$  characteristics is a straight line.  $T_{sh}/I_a$  is shown by dotted line.

**Conclusion:** Since  $T \propto I_a$ , therefore its starting torque is less than that of series motor. Hence it is used where starting torque required is not high, i.e. lathe machine tools etc.

 **$N/I_a$  Characteristics**

$E_b \propto \phi N$ . Since  $\phi$  is constant, and  $E_b$  is also practically constant, therefore the speed of a D.C. shunt motor is constant. But  $E_b$  decreases with increase in load current,  $N \propto E_b$  or  $N \propto (V - I_a R_a)$



therefore speed of motor falls. But at the same time, flux  $\phi$  decreases due to armature reaction, and since  $N \propto 1/\phi$ , therefore speed tries to increase. Fall in speed due to decrease in  $E_b$  is more than the increase in speed due to decrease in flux due to armature reaction, the characteristics is slightly dropping.

**Conclusion:** Shunt motor is regarded as constant speed motor and is used to drive loads having constant speed, i.e. lathe machine loads etc.

 **$N/T_a$  Characteristics**

The shape of this curve is similar to that of  $N/I_a$ .

**COMPARISON BETWEEN SERIES AND SHUNT MOTORS**

Series Motor	Shunt Motor
High starting torque $T \propto I_a^2$	Low starting torque $T \propto I_a$
Low speed at high loads and high speeds at low loads. Not to be used at no load.	Constant speed motor.

**APPLICATION OF D.C. MOTORS**

Type of Motor Characteristics	Application
Shunt	<ul style="list-style-type: none"> <li>Approximately constant speed</li> <li>Adjustable speed</li> <li>Medium speed</li> </ul> For driving constant speed line shafting lathes, centrifugal pumps, machine tools, blowers and fans and reciprocating pumps
Series	<ul style="list-style-type: none"> <li>Variable speed</li> </ul> For traction work, i.e. electric



	<ul style="list-style-type: none"> <li>• Adjustable varying speed</li> <li>• High starting torque</li> </ul>	locomotives, rapid transit systems, trolley cars etc., cranes and hoists conveyors.
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**Example**

*A D.C. series motor of resistance 1 ohm between terminals runs at 800 rpm at 200 V with a current of 15 amps. Find the speed at which it will run when connected in series with a 4 ohm resistance and taking the same supply voltage.*

**Solution**

Total armature resistance  $R = R_a + R_{se} = 1$  ohm

$$E_{b1} = V - IR = 200 - 15 \times 1 = 185V$$

In the second case, control resistance is 4 ohms

Total armature resistance = 4 + 1 = 5 ohms

Therefore,  $E_{b2} = V - IR = 200 - 15 \times 5 = 125$  V

Now, 
$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} \text{ or } N_2 = \frac{E_{b2}N_1}{E_{b1}} = \frac{125(800)}{185} = 540.5 \text{ r.p.m.}$$

**Example**

*A 10 H.P. 230 V shunt motor takes an armature current of 6A from 230 V mains at no load runs at 1200 r.p.m. The armature resistance is 0.25 Ω. Determine speed and electromagnetic torque when the armature takes 36 amps. with the same flux.*

**Solution**

No load  $E_{b1} = V - I_{a0}R_a = 230 - 6 \times 0.25 = 228.5$  volts

Back e.m.f. when loaded  $E_{b2} = 230 - 36 \times 0.25 = 221$  volt.

$$E_b \propto \phi N$$

But for shunt motor

$$E_b \propto N$$

Hence 
$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} \text{ or } N_2 = \frac{E_{b2}N_1}{E_{b1}} = \frac{221 \times 1200}{228.5} = 1161 \text{ r.p.m.}$$

Electromagnetic torque when armature takes 36 amps.

$$= \frac{E_b I_a}{2\pi(N/60)} = \frac{221 \times 36}{2\pi \times \frac{1161}{60}} = 65.44 \text{ Nm}$$

---

A 240 volts shunt motor runs at 850 r.p.m. when the armature current is 70 amperes. The armature resistance is 0.1 ohm. Calculate the required resistance to be placed in series with the armature to reduce the speed to 650 r.p.m. when the armature current is then 50 amperes.

### Solution

---

We know that for a d.c. motor

$$E_b = V - I_a R_a = \phi ZNP / 60A E$$

$$240 - 70 \times 0.1 = \phi.Z.850.P/60.A = K.850$$

$$K = 233/850$$

Now for motor to run at 650 r.p.m., we have

$$240 - 50 \times R_a = K \times 650 = 233 \times 650/850 = 178.17$$

$$\therefore R_a = 1.236 \Omega$$

$$\therefore \text{Resistance to be put in series with the armature} = 1.236 - 0.1 = 1.136 \Omega$$

## ASSIGNMENT

### D.C. MACHINES/GENERATORS

**Q.1. (AMIE W12, 4 marks):** Name the parts of a d.c. machine.

**Q.2. (AMIE W06, 10, S10, 12 marks):** Draw a 4-pole d.c. machine/generator construction diagram labelling its main parts. describe briefly its (i) Field system (ii) Armature (iii) Commutator (iv) Brushes.

**Q.3. (AMIE W07, 13, 5 marks):** Derive the equation for induced emf of a d.c. machine. Which are the quantities in the expression constant?

**Q.4. (AMIE W07, 8 marks):** What are different types of d.c. generators and d.c. motors? Discuss with schematic diagrams.

**Q.5. (AMIE W13, 4 marks):** Differentiate between series and shunt d.c. generators.

**Q.6. (AMIE S09, 8 marks):** Draw the load characteristics of various types of d.c. generators in one figure, assuming same no-load terminal voltage. Compare these characteristics Give one application of each generator.

**Q.7. (AMIE W11, 4 marks):** Draw characteristics of d.c. shunt generator.

**Q.8. (AMIE S13, 6 marks):** What are compensating windings and inter poles in a d.c. machine and why are they used?

**Q.9. (AMIE W06, 8 marks):** A 6 pole lap wound d.c. generator has an armature with 90 slots and 6 conductors per slot and rotates at speed of 6000 rpm. The flux per pole is 100 mWb. calculate the emf produced.

Answer: 540 V

**Q.10. (AMIE S13, 6 marks):** The armature of a four pole d.c. generator has 47 slots, each containing six conductors. The armature winding is wave connected and the flux per pole is 25 mWb. At what speed must the machine be driven to generate an emf of 250 V?

Answer: 1064 rpm

**Q.11. (AMIE W07, 10 marks):** A 4 pole, 220 V. d.c. shunt generator has an armature resistance of 1 ohm, shunt field resistance of 220 ohms. The generator supplies power to a 10 ohm resistor. Calculate the generated emf of the generator, if the load voltage is to be maintained at 220 V. Assume brush contact drop to be 2 V.

Answer: 245 V

**Q.12. (AMIE W10, 4 marks):** An 8 pole lap wound d.c. generator has 90 conductors, a flux of 40 mWb per pole and is driven at 400 rpm. Find open circuit voltage. Derive the expression used.

**Q.13. (AMIE S09, 12 marks):** The induced emf in a d.c. machine while running at 750 rpm is 220 V. Calculate the (i) speed at which the induced emf will be 250 V, assuming constant flux, and (ii) percentage increase in the field flux for an induced emf of 250 V and speed of 700 rpm.

**Q.14. (AMIE W09, 8 marks):** A belt driven shunt generator delivers 110 kW to a 220 V bus bar while running at 500 rpm. The belt breaks suddenly, but the machine continues to run as a motor, drawing 11 kW from the bus bar. Assuming armature and field resistances to be 0.025  $\Omega$  and 55  $\Omega$ , respectively and total brush voltage drop to be 1 V, determine the speed at which the machine shall run under this changed situation. Ignore armature reaction.

Answer: 462 rpm

**Q.15. (AMIE W12, 8 marks):** A shunt d.c. generator has an open circuit no load voltage of 127 V and a voltage of 120 V on full load. The armature resistance is 0.02 ohm. Find armature current, field current and load current. Field resistance is 15 ohm.

Answer: 350 A, 8 A, 342 A

**Q.16. (AMIE W10, 4 marks):** Name different types of d.c. motors.

**Q.17. (AMIE W08, S11, 10 marks):** Explain what is meant by back e.m.f. Explain the principle of torque production in a d.c. motor.

**Q.18. (AMIE S07, 12, W09, 5 marks):** Derive the torque equation of a d.c. motor.

**Q.19. (AMIE S10, 10 marks):** Differentiate between series and shunt d.c. motors as regards their construction and characteristics.

**Q.20. (AMIE W10, 4 marks):** Name some applications of single phase motor.

**Q.21. (AMIE W07, 6 marks):** A 6-pole lap-wound d.c. shunt motor is fed from 400 V d.c. supply. The armature current is 50 A and armature resistance is 0.2 ohm. If flux per pole is 0.05 Wb and the motor has 540 conductors find the speed.

Answer: 860 rpm

**Q.22. (AMIE W13, 8 marks):** A 60 hp 230 V d.c. shunt motor has an armature resistance of 0.05  $\Omega$  and field circuit resistance of 46  $\Omega$ . The no load speed is 1000 rpm. Find the speed when the line current is 75 A. Assume that the motor has compensate winding.

Answer: 986 rpm